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2010 J. Phys.: Condens. Matter 22 155101

(http://iopscience.iop.org/0953-8984/22/15/155101)

J. Phys.: Condens. Matter **22** (2010) 155101 (5pp) doi:10.1088/0953-8984/22/15/155101

Depinning dynamics of two-dimensional magnetized colloids on a random substrate

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Received 6 November 2009, in final form 29 December 2009
Published 9 March 2010
Online at stacks.iop.org/JPhysCM/22/155101

Abstract

We perform Langevin simulations on the depinning dynamics of two-dimensional magnetized colloids on a random substrate. On increasing the magnetic field strength, we find for the first time a crossover from plastic to smectic flows as well as a crossover from smectic to elastic crystal flows above depinning. For both the smectic and elastic crystal flows, a power-law scaling relationship could be obtained between the average velocity and applied driving force. The scaling exponent is found to be larger than 1 for smectic flow. But, for the elastic crystal flow, the scaling exponent is found to be less than 1. For the plastic flow, no power-law scaling relationship between the average velocity and applied driving force can be derived and history dependence of the depinning occurs. Within the crossover from plastic to smectic flows, a sudden decrease in the critical driving force is observed, and a sudden increase is found in the critical driving force across the crossover from smectic to elastic crystal flows, accompanied by a crossing of the curves of average velocity versus driving force.

1. Introduction

Colloids can display intriguing phase transitions which are ubiquitous in nature and have been studied for decades [1]. Recently much attention has been paid to colloidal systems [2–27]. Since their size and nature allow us to easily tune the shape and strength of the colloidal interaction and to directly visualize the particles under a microscope [9, 11, 13, 16, 19, 20, 25–27], colloids provide an ideal model system for studying the basic problems in condensed matter physics, in particular, for studying two-dimensional (2D) ordering and melting [9, 19–22, 25–27].

Most recently, the response of colloidal system under an external field has attracted extensive attention. One can obtain significant insight into the nature of order by investigating the response of a system to a perturbing force, and through studying the colloidal dynamics, one can control and tailor the external perturbation [3] as well. In 2002, the depinning of 2D charged colloids on a toy substrate was investigated numerically; a crossover from elastic crystal to plastic flows was found above depinning [4]. Then in 2003, we revisited the dynamics of 2D charged colloids on a realistic quenched substrate, and the crossover from elastic crystal to plastic flows was also observed above the depinning [7]. Then in 2008, the crossover from elastic crystal to plastic flows was confirmed experimentally in 2D charged colloidal crystals [19].

Some colloidal particles are superparamagnetic; once a magnetic field is applied, the magnitude of the induced magnetic moment in the particles scales linearly with the magnetic field strength [3, 12–14, 20, 22, 25–27]. Therefore, an almost perfect 2D system could be realized and the order and dynamics of 2D systems could be established easily in magnetized colloids. However, up to now, the dynamics of driven magnetized colloids has remained open.

In 2008, we attempted an investigation of the dynamics of 2D magnetized colloids on a realistic quenched substrate [23]. On increasing the strength of the substrate, we found a crossover from elastic crystal to plastic flows above depinning, in agreement with that of charged colloids [4, 7, 19]. But, in the plastic regime, a crossover from plastic to smectic flows as well as a crossover from smectic to elastic crystal flows was observed on increasing the applied driving force above depinning.

Because the interaction between magnetized colloidal particles is proportional to the square of the magnitude

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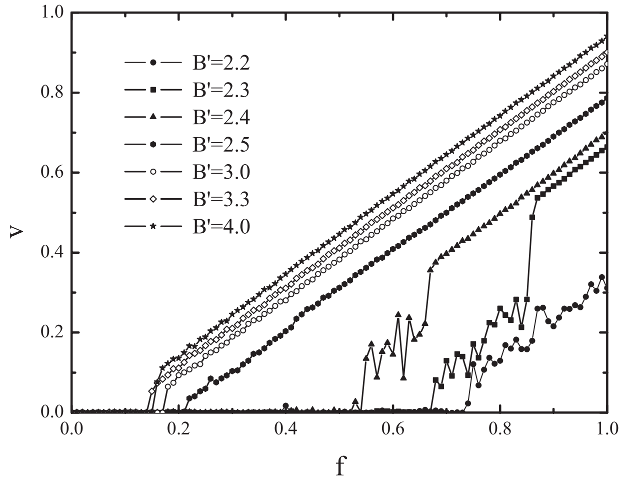

Figure 1. Series of curves of average velocity v versus applied driving force f for different values of B'.

of the magnetic moment [3, 12–14, 20, 22, 25–27], the dynamics of magnetized colloids is completely controlled by the external magnetic field strength. This is valuable for studying the dynamics as well as realizing macroscopic separation of different species of particles in biological and other mesoscopic systems by adjusting the magnetic field systematically.

In this paper, we will investigate the depinning of 2D magnetized colloids on a realistic quenched substrate while changing the magnetic field strength systematically. A crossover from plastic to smectic flows as well as a crossover from smectic to elastic crystal flows will be found above the depinning with increase in the magnetic field strength.

2. The model

The motion of magnetized colloidal particles is described by the Langevin equation [4, 7, 23, 28],

$$\eta\frac{\mathrm{d}\mathbf{R}_i}{\mathrm{d}t} = -\sum_{i\neq j}\nabla_i U_{\mathrm{cc}}(\mathbf{R}_i-\mathbf{R}_j) - \sum_{j'}\nabla_i U_{\mathrm{cs}}(\mathbf{R}_i-\mathbf{r}_{j'}) + \mathbf{f}_i^{\mathrm{L}}(t) + \mathbf{f}, \tag{1}$$

where η is a damping constant, chosen as unity in this paper. $\mathbf{R}_i$ and $\mathbf{r}_{j'}$ are the coordinates of the ith colloid and the j'th center of pinning in the substrate, respectively. U_{cc} is the potential of interaction between colloids, U_{cs} is the potential of interaction between the colloid and pinning in the substrate, $\mathbf{f}$ is the driving force due to an applied electric field [19]. $\mathbf{f}_i^{\mathrm{L}}$ is the Langevin random fluctuating force, which is described by [7, 23, 28] $\langle\mathbf{f}_i^{\mathrm{L}}(t)\rangle = 0$ and $\langle f_{i\alpha}^{\mathrm{L}}(t)f_{j\beta}^{\mathrm{L}}(t')\rangle = 2\eta T\delta_{ij}\delta_{\alpha\beta}\delta(t-t')$, where the temperature T is fixed as the bare Kosterlitz–Thouless melting temperature of the 2D system [29]. Here, we neglect the hydrodynamic interaction between the colloidal particles since we are in the low volume fraction area.

The potential of interaction between magnetized colloids is that of parallel dipoles caused by a magnetic field perpendicular to the plane of the substrate, and reads as [3, 23, 27]

$$U(r) = \frac{\mu_0}{4\pi}\frac{M^2}{r^3}, \tag{2}$$

where μ_0 is the magnetic permeability of free space. The magnetic moment $M = \chi B$, with χ the effective susceptibility and B the magnetic field strength. Here we take $\frac{\mu_0}{4\pi}M^2 = B'$ as the dimensionless strength of interaction between colloids, relatively to the pinning strength in the substrate. B' can be externally controlled by means of the magnetic field strength B [26]. We change B' (i.e., change the magnetic field strength

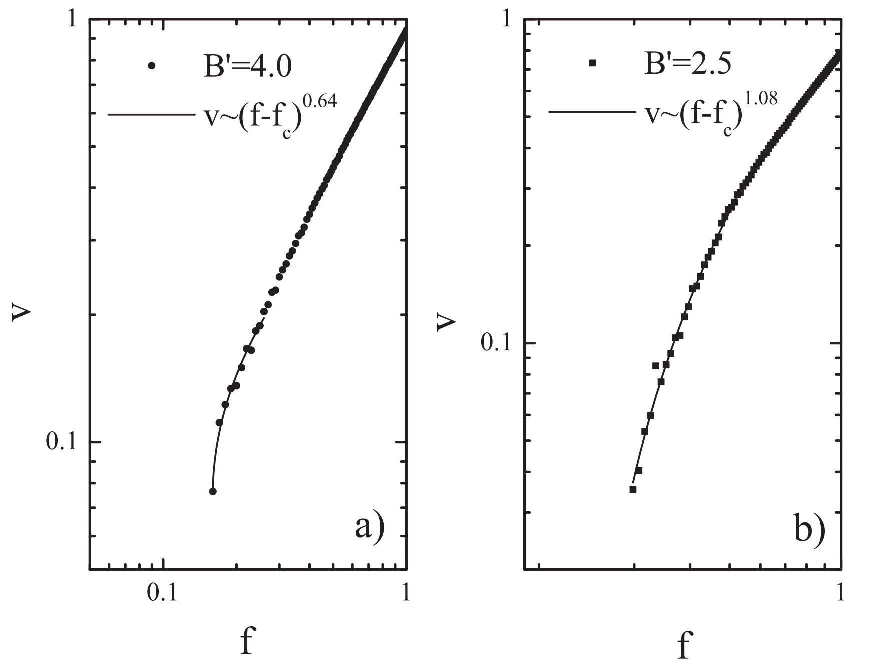

Figure 2. The power-law scaling relationship between v and f above depinning. (a) and (b) are for $B' = 4.0$ and 2.5, respectively.

B) systematically so as to investigate the depinning of 2D magnetized colloids.

As in [23], the substrate is chosen as a realistic quenched one and is simulated by randomly distributed point-like pinning centers. The interaction between colloids and pins in the substrate is generally modeled as a attracting Gaussian potential [7, 23]. The pinning strength in the substrate is fixed in this paper.

All lengths are measured in units of the lattice constant a_0 of the ideal triangular lattice. 400 magnetized colloids are placed initially in a perfect triangular lattice with periodic boundary conditions, and 800 point-like pinning centers are randomly distributed in the substrate. The driving force $\mathbf{f}$ is increased along the horizontal symmetry axis x and the average velocity $v = \frac{1}{N}\sum_i^N \mathbf{v}_i \cdot \hat{\mathbf{x}}$, where N is the number of magnetized colloids, is measured at each increment in this direction.

3. Results and conclusions

Figure 1 presents a series of curves of average velocity v versus applied driving force f for different values of B'. It is found that, for each curve, there exists a critical driving force f_c below which the colloids are pinned and small advances due to fluctuations can be neglected and the average velocity v is zero. Above f_c, v increases with f and we find different features in the curves for different values of B'. For small values of B' (see the curves for $B' = 2.2$–2.4), v increases with f non-linearly; even steps appear above f_c. No power-law scaling relationship could be derived between v and f. These are typical characteristics of plastic flow in velocity–force dependence, consistent with our previous simulations on charged colloids [7] and magnetized colloids [23] as well as vortex lattices [28]. It should be noted that it may be that the finite size of the system leads to such strong fluctuations that a scaling fit could not be made. It is possible that if much larger systems are simulated the fluctuations will be averaged out and a power-law fit with an exponent of 2.0 might be obtained [4].

For larger values of B' (see the curves for $B' = 2.5$–3.3), steps disappear and linearity begins to occur in the curve of v versus f above f_c. A power-law scaling relationship could be obtained between v and f above f_c. The features are the characteristics of elastic flow. We find that the scaling exponent is larger than 1, as seen in figure 2(b) where we give the power-law scaling relationship between v and f above f_c for $B' = 2.5$ and the scaling exponent $\zeta = 1.08$ is found.

When B' is increased above a certain value ($B' \geqslant 3.3$), a more evident linearity is found in the v–f dependence above f_c and the power-law relationship could also be obtained. But, in this case, the scaling exponent is found to be less than 1, as shown in figure 2(a) where we present the power-law scaling relationship between v and f near depinning for $B' = 4.0$ and the scaling exponent $\zeta = 0.64$ is observed. This is very close to those of 2D charged colloids [19] and 2D charge density wave (CDW) systems [30, 31] as well as Wigner crystal [32] where $\zeta = 2/3$.

Figure 3 presents the history dependence of the depinning processes for different values of B'. An evident history dependence is found for small values of B', as seen in the upper panel where the history dependence of the depinning is given for $B' = 2.4$. This is a characteristic of plastic flow. But for large enough values of B', the history dependence disappears, as shown in the two lower panels where we give the history dependence of depinning for $B' = 3.0$ and 3.5, coincident with the characteristics of elastic flow.

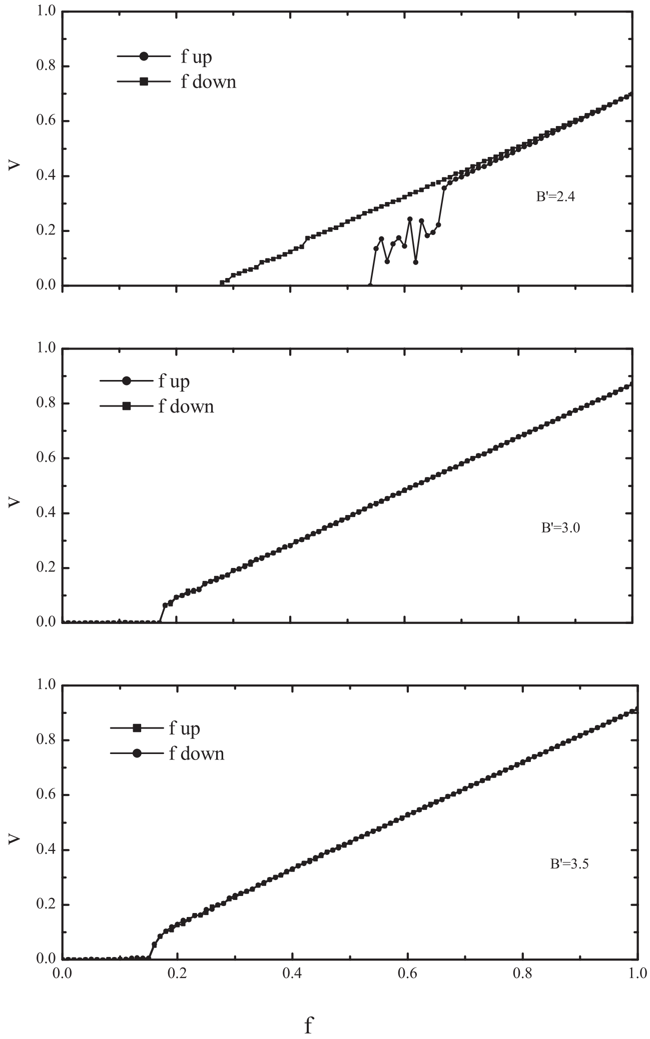

Figure 3. History dependence of the depinning for $B' = 2.4$, 3.0, and 3.5.

To distinguish the above flows explicitly, in figure 4 we give the colloidal coordinates (figures 4(a)–(c)) and corresponding static structure factors (figures 4(d)–(f)) at a certain driving force above f_c ($f/f_c = 1.1$) for $B' = 2.4$ ((a) and (d)), 2.5 ((b) and (e)), and 3.3 ((c) and (f)). The structure factor is defined as $S(\mathbf{k}) = \langle|\frac{1}{N}\sum_{i=1} e^{i\mathbf{k}\cdot\mathbf{R}_i}|^2\rangle$. For small values of B' ($B' \leqslant 2.4$), colloids flow plastically. In such a case, the lattice is destroyed and the six neighbors of the lattice cannot be maintained as they move, as seen in figure 4(a). No Bragg peaks appear and only one peak is found, at the center ($k_x = k_y = 0$), as shown in figure 4(d).

For larger values of B' ($B' = 2.5$–3.3), colloids move in a smectic phase above depinning. In this case, although the lattice is distorted, each colloidal particle keeps the same

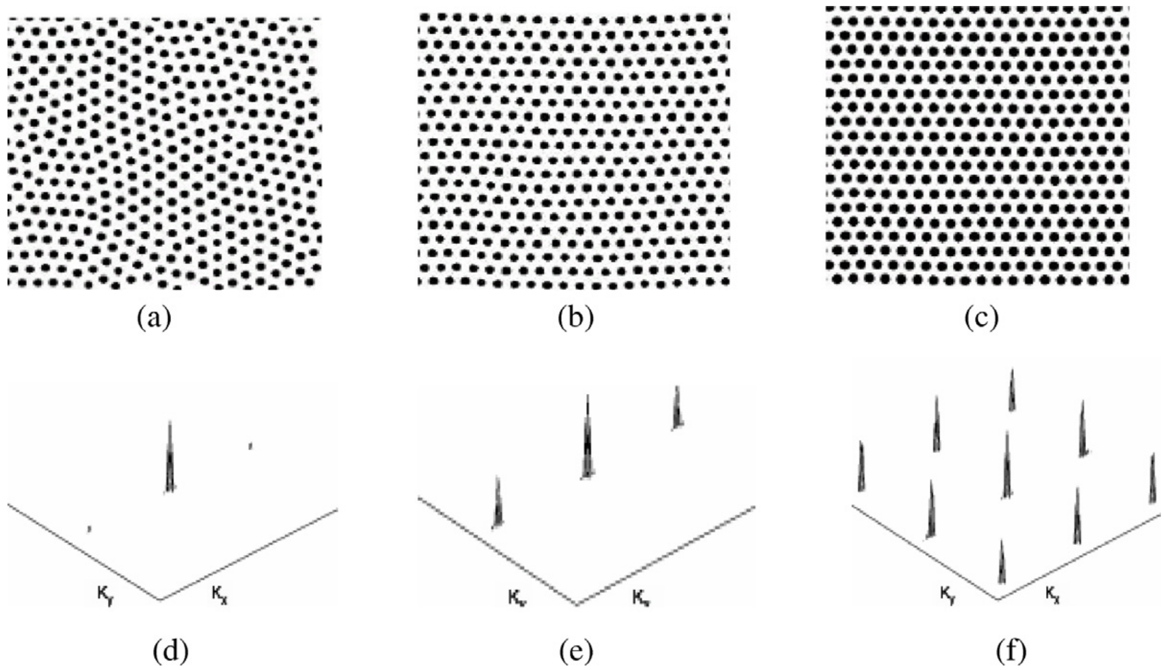

Figure 4. Colloid coordinates ((a)–(c)) and structure factors ((d)–(f)) at a driving force above depinning ($f/f_c = 1.1$). (a) and (d) are for $B' = 2.4$, (b) and (e) are for $B' = 2.5$, (c) and (f) are for $B' = 3.3$.

six neighbors as it moves, as shown in figure 4(b). The corresponding structure factor shows two Bragg peaks along the driving force direction, as given in figure 4(e).

When B' is increased above certain values ($B' \geqslant 3.3$), elastic crystal flow takes place above depinning, colloids move in a perfect lattice and all the colloids keep the same six neighbors as they move; see figure 4(d). The six Bragg peaks appear clearly in the corresponding structure factor, as seen in figure 4(f).

In addition, we find that, accompanying the crossover from the plastic to smectic flows with increase in B', the critical driving force f_c decreases suddenly, as shown in figure 5(b) where we give the curve of f_c versus B'. Corresponding to the sudden decrease, a negative peak is shown in the differential $\mathrm{d}f_c/\mathrm{d}B'$, as presented in the inset of figure 5(b).

However, with a further increase in B', a crossover from smectic to elastic crystal flows takes place, and within the crossover from smectic to elastic crystal flows, a distinct characteristic is found: that f_c increases suddenly, as seen in figure 5(a). The differential $\mathrm{d}f_c/\mathrm{d}B'$ displays a peak, as shown in the inset of figure 5(a), and corresponding to the peak, a clear crossing of v versus f curves takes place. From the curves for $B' = 3.3$ and 4.0 in the figure 1, we can see this point.

To summarize, we have investigated the depinning of two-dimensional magnetized colloids. On increasing the magnetic field strength, we find for the first time a crossover from plastic to smectic flows as well as a crossover from smectic to elastic crystal flows above depinning. A power-law scaling relationship could be obtained between the average velocity and the applied driving force above depinning for both the elastic crystal and smectic flows, but the scaling exponents are found to be quite different. For the elastic crystal flow, the scaling exponent is less than 1, but it is larger than 1 for the smectic flow. Accompanying each crossover, the critical

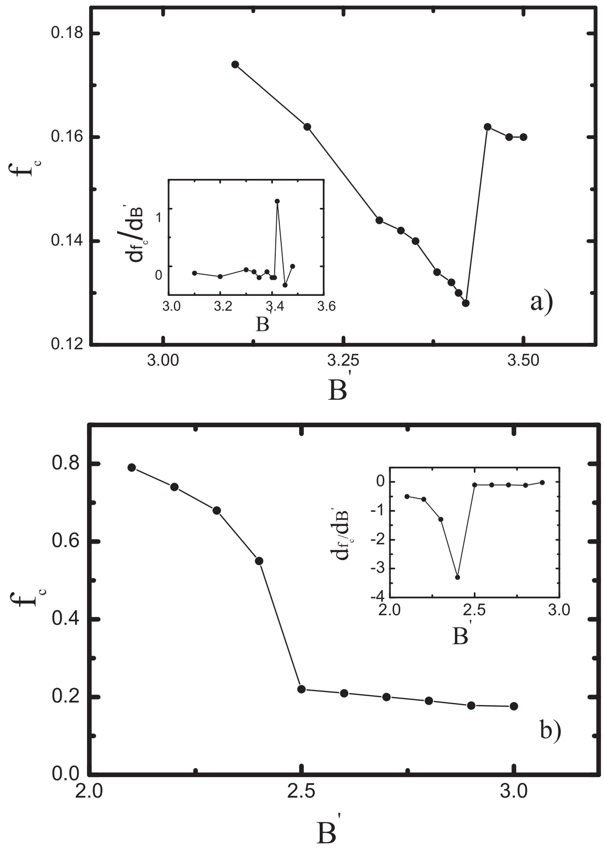

Figure 5. Critical driving force f_c versus B'. The insets show the corresponding differential $\mathrm{d}f_c/\mathrm{d}B'$ versus B'.

driving force shows a sudden change. It should be pointed out that the transition to a line ordering is well known in concentrated suspensions of monodispersed spheres from both simulations [6, 32, 33] and experiments [22, 34, 35]. In fact, the existence of a smectic phase was predicted by Nelson and Halperin [36, 37] as a phase intermediate between the isotropic liquid and crystalline phase with two continuous defect-mediated transitions [22]. Our results are helpful for investigating the response of colloidal systems under magnetic and electric fields as well as for controlling and tailoring the external fields according to colloidal dynamics.

Acknowledgment

The work was supported by the Foundation of Henan Educational Committee under account 2008A140011.

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